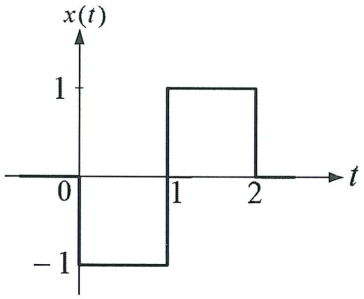
EE 362: Signals and Systems Name: Solution

Spring 2019

Homework #6 (Due: Thursday, March 7, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

(1) Use Fourier properties to compute the Fourier transform of the following signal. [6 points]



(2) A continuous-time signal $x(t)$ has Fourier transform

$$X(\omega) = \frac{1}{j\omega + b}$$

where b is a constant. Determine Fourier transform $V(\omega)$ of the following signals: [2 points each; 10 points total]

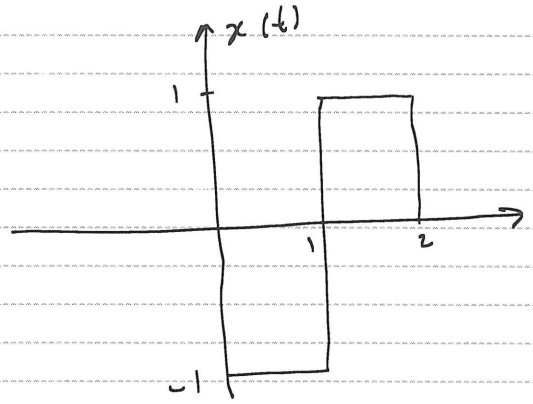
(a) $v(t) = x(2t)$
(b) $v(t) = tx(t)$
(c) $v(t) = e^{-j10t}x(t)$
(d) $v(t) = x(t)\sin(15t)$
(e) $v(t) = x(t+3)$

(3) A signal has Fourier transform as, [6 points]

$$X(\omega) = \frac{1}{j}\left[\text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{2}\right) - \text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{2}\right)\right]$$

Compute $x(t)$

(1)



$\Rightarrow -P_1(t-0.5) + P_1(t-1.5)$

$\therefore x(t) = -P_1(t-0.5) + P_1(t-1.5)$

$P_\tau(t) \longleftrightarrow \tau \, \text{sinc}\left(\frac{\tau\omega}{2\pi}\right)$

$P_\tau(t-c) \longleftrightarrow \tau \, \text{sinc}\left(\frac{\tau\omega}{2\pi}\right) e^{-j\omega c}$

$\therefore x(t) \longleftrightarrow -\text{sinc}\left(\frac{\omega}{2\pi}\right) e^{-j\omega/2} + \text{sinc}\left(\frac{\omega}{2\pi}\right) e^{-j3\omega/2}$

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(2) (a) $v(t) = x(2t) \Rightarrow V(\omega) = \frac{1}{2}\left(\frac{1}{j\frac{\omega}{2} + b}\right)$

(b)

$v(t) = t\,x(t)$

Use multiplication by a power of t property

$t^n x(t) \longleftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega)$, $n = 1, 2, \ldots$

In our case $n = 1$

$\therefore t\,x(t) \longleftrightarrow j\frac{d}{d\omega} X(\omega)$

$\therefore V(\omega) = j\frac{d}{d\omega}\left(\frac{1}{j\omega + b}\right) = j\frac{-j}{(j\omega+b)^2}$

$\therefore V(\omega) = \frac{1}{(j\omega+b)^2}$

(c)

$$V(t) = e^{-j10t}\, x(t)$$

Use multiplication by complex exponential property

$$x(t)\, e^{j\omega_0 t} \longleftrightarrow X(\omega - \omega_0) \quad / \omega_0 \text{ is real number}$$

$$x(t)\, e^{-j\omega_0 t} \longleftrightarrow X(\omega + \omega_0)$$

$$\therefore V(\omega) = X(\omega + 10) = \frac{1}{j(\omega+10)+6}$$

(d) $V(t) = x(t)\sin(15t)$

$$x(t)\sin(\omega_0 t) \longleftrightarrow \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$$

$$\therefore V(\omega) = \frac{j}{2}\left[X(\omega+15) - X(\omega-15)\right]$$

$$\therefore V(\omega) = \frac{j}{2}\left[\frac{1}{j(\omega+15)+6} - \frac{1}{j(\omega-15)+6}\right]$$

(e) $V(t) = x(t+3)$

$$\therefore V(\omega) = \left(\frac{1}{j\omega+6}\right) e^{j3\omega}$$

(3)

$$X(\omega) = \frac{1}{j}\left[\text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{2}\right) - \text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{2}\right)\right]$$

$$X(\omega) = -j\left[\text{sinc}\left(\tfrac{2\omega}{\pi} - \tfrac{1}{2}\right) - \text{sinc}\left(\tfrac{2\omega}{\pi} + \tfrac{1}{2}\right)\right]$$

$$= j\left[\text{sinc}\left(\tfrac{2\omega}{\pi} + \tfrac{1}{2}\right) - \text{sinc}\left(\tfrac{2\omega}{\pi} - \tfrac{1}{2}\right)\right]$$

From multiplication by sinusoid property

$$x(t)\sin(\omega_0 t) \leftrightarrow \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right] \quad -①$$

Also, we know that

$$P_\tau(t) \leftrightarrow \tau\,\text{sinc}\left(\frac{\tau\omega}{2\pi}\right)$$

If we choose τ to be 4:

$$P_4(t) \leftrightarrow 4\,\text{sinc}\left(\frac{2\omega}{\pi}\right)$$

From ① we can get

$$P_4(t)\sin\left(\tfrac{\pi}{4}t\right) \leftrightarrow \frac{j}{2}\left[4\,\text{sinc}\left(\frac{2(\omega+\frac{\pi}{4})}{\pi}\right) - 4\,\text{sinc}\left(\frac{2(\omega-\frac{\pi}{4})}{\pi}\right)\right]$$

$$\leftrightarrow 2j\left[\text{sinc}\left(\tfrac{2\omega}{\pi} + \tfrac{1}{2}\right) - \text{sinc}\left(\tfrac{2\omega}{\pi} - \tfrac{1}{2}\right)\right]$$

↑ we need to divide by 2

Then

$$\frac{1}{2}P_4(t) \leftrightarrow 2\,\text{sinc}\left(\frac{2\omega}{\pi}\right)$$

From ① we can get:

$$\frac{1}{2}P_4(t)\sin\left(\tfrac{\pi}{4}t\right) \leftrightarrow j\left[\text{sinc}\left(\tfrac{2\omega}{\pi} + \tfrac{1}{2}\right) - \text{sinc}\left(\tfrac{2\omega}{\pi} - \tfrac{1}{2}\right)\right]$$

$$\therefore x(t) = \frac{1}{2}P_4(t)\sin\left(\frac{\pi}{4}t\right)$$